NUMERICAL MODELING OF THE STATIONARY ELECTRON DISTRIBUTION FUNCTION IN A WEAKLY IONIZED GAS IN NONUNIFORM ELECTRIC FIELDS

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To study a number of problems in the physics of gas discharges (the structure of the regions near the electrodes, motionless striations, etc.) a kinetic approach must be used to describe the motion of the electrons. When spatial variables are present the most effective method of numerical modeling is the Monte Carlo method. However the statistical error of this method decreases comparatively slowly ($\sim N_t^{-1/2}$) as the number of trajectories N_t followed is increased, and significant amounts of computer time are required to solve the kinetic equation, especially for weak fields E. The total transport scattering cross section σ_t is usually many times greater than the inelastic scattering cross section σ, and $f_0(\varepsilon, r)$ – the electron energy distribution function (EEDF) – can be found with adequate accuracy on the basis of the two-term approximation (TA) [1]

$$-\left(\frac{\partial}{\partial \mathbf{r}} - e\mathbf{E}\frac{\partial}{\partial \varepsilon}\right)\frac{v^2}{3N\sigma_t}\left(\frac{\partial}{\partial \mathbf{r}} - e\mathbf{E}\frac{\partial}{\partial \varepsilon}\right)f_0 = \mathrm{st}\{f_0\}. \tag{1}$$

Here v is the velocity of the electrons; N is the concentration of atoms; $\mathrm{st}\{f_0\}$ is a collisional term describing the exchange of energy in electron–atom collisions; and, collisions between electrons are neglected. The correctness of using the TA in a wide range of values of E/N in different gases was demonstrated for spatially uniform situations in a number of works (see, for example [2-4]), where the results were compared with the solution of the complete kinetic equation. Although the diffusion-drift model, following from Eq. (1), for describing the motion of the electrons is the basis for the study of gas discharges, the error of the TA in nonuniform fields has never been analyzed. Only the behavior of the EEDF in a uniform field near an ideally absorbing anode in a model (no inelastic collisions) gas has been studied [5, 6]. The TA is formally applicable at large ($N\sigma_t x \gg 1$) distances away from electrodes under the condition $N\sigma_t \ell \gg 1$ (ℓ is the characteristic distance over which the electric field intensity changes). The calculations of the EEDF performed below for neon show that the TA is actually applicable in a much wider range. In this paper the choice of boundary conditions for Eq. (1) is also analyzed and an efficient algorithm for calculating the solution for Eq. (1) is presented.

Boundary Conditions. The capabilities of modern computers make it necessary to limit the analysis to the one-dimensional case, for example, the behavior of the EEDF between infinite plane-parallel electrodes. We shall study the situation when the reflection of electrons from the electrodes is insignificant; this problem is more difficult from the viewpoint of the applicability of the TA. Then the following boundary conditions are imposed on the electron distribution function $f(\varepsilon, \mu, x)$:

$$f(\varepsilon,\ \mu > 0,\ x = 0) = f_i(\varepsilon,\ \mu),\ f(\varepsilon,\ \mu < 0,\ x = d) = 0, \tag{2}$$

where μ is the cosine of the angle between the direction of the electric field and the electron velocity; f_i is the distribution function of the electrons injected from the cathode; the point x = 0 corresponds to the cathode and x = d corresponds to the anode. On the basis of the TA we have

$$f(\varepsilon, \mu, x) = \frac{1}{2}f_0(\varepsilon, x) + \frac{3}{2}\mu f_1(\varepsilon, x),\quad f_1 = -\frac{1}{3N\sigma_t}\left(\frac{\partial}{\partial x} - eE\frac{\partial}{\partial \varepsilon}\right)f_0,$$

and the conditions (2) can be satisfied only approximately. For this reason the choice of boundary conditions for Eq. (1) is somewhat arbitrary. The most commonly employed boundary conditions are Marshak's

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$$\int_0^1 \mu f(\varepsilon, \mu, x=0)\,d\mu = \int_0^1 \mu f_i(\varepsilon, \mu)\,d\mu, \quad \int_{-1}^0 \mu f(\varepsilon, \mu, x=d)\,d\mu = 0 \tag{3}$$

and Mark's

$$f(\varepsilon, \mu=\mu_*, x=0) = f_i(\varepsilon, \mu=\mu_*), \quad f(\varepsilon, \mu=-\mu_*, x=d) = 0 \tag{4}$$

($\mu_* = 1/\sqrt{3}$ [7]). The expression (4) with $\mu_* = 1$ was employed in [5, 6]. Our calculations show, however, that this choice of μ_* gives a higher error than the best of the conditions (3) and (4). Let the angular distribution of the injected electrons follow the cosine law $f_i(\varepsilon, \mu) = 3\mu f_i(\varepsilon)$, which is true, for example, for the case of ion–electron emission [8]. Then the expressions (3) and (4) assume the form

$$\begin{aligned} C_0 f_0(\varepsilon, x=0) + C_1 f_1(\varepsilon, x=0) &= f_i(\varepsilon), \\ C_2 f_0(\varepsilon, x=d) - f_1(\varepsilon, x=d) &= 0, \end{aligned} \tag{5}$$

where $C_1 = 1/2$; for Marshak's conditions $C_0 = 1/4$, $C_2 = C_1$; for Mark's conditions $C_0 = 1/2 \times \sqrt{3}$, $C_2 = 1/\sqrt{3}$. When $\sigma_t \gg \sigma$ in a uniform electric field the average energy of the electrons is much higher than the energy $\varepsilon_* = eE/N\sigma_t$, acquired over the mean-free path length. For this reason the electron velocity distribution function is nearly spherically symmetric, which is what determines the good accuracy of the TA. If the energy of the injected electrons is less than ε_*, then near the cathode electrons move predominantly along the direction of the electric field and the TA overestimates the value of the transmission coefficient of the electrons. An analogous situation arises with large gradients of the field in the region near the anode, into which low-energy electrons flow. The error of the TA can be reduced by selecting the boundary conditions (5) with variable values of C_1 and C_2, depending on the ratio $\kappa = \varepsilon_*/\varepsilon$. It is not difficult to show that for $\kappa \gg 1$, we must set $C_1 = C_2 = 1$. For $\kappa \ll 1$, Marshak's conditions are best in some strict sense [7]. The simplest interpolation between these limiting cases

$$C_1 = C_2 = \frac{1}{2}\left(1 + \frac{\kappa}{1+\kappa}\right), \quad C_0 = \frac{1}{4} \tag{6}$$

already makes it possible to obtain satisfactory accuracy for the TA in a wide range of values of E/N and $N\sigma_t\ell$.

Algorithm for the Numerical Solution. Solving Eq. (1) is a quite laborious procedure. In many cases, however, the intensity of the electric field in the interelectrode gap does not change sign, and it is possible to construct an efficient algorithm for solving (1) numerically. The temperature of the electrons is usually much higher than the temperature of the gas, and the heating of electrons in elastic collisions with atoms and in collisions of the second kind can be neglected. Then, after making the substitution of variables $\varepsilon = W + e\varphi$ (φ is the potential of the electric field) Eq. (1) and the boundary conditions (5) can be written in the form

$$\begin{aligned} &-\frac{\partial}{\partial x}\frac{v^2}{3N\sigma_t}\frac{\partial f_0}{\partial x} - \frac{\partial}{\partial W}\frac{2m}{M}Nv^2\sigma_t\varepsilon f_0 + \nu f_0 = \\ &= 4\nu_i(2\varepsilon + I) f_0(2W + e\varphi + I, x) + \sum_k \nu_k (W + \varepsilon_k) f_0(W + \varepsilon_k, x), \\ &C_0 f_0 - \frac{C_1}{3N\sigma_t}\frac{\partial f_0}{\partial x}\bigg|_{x=0} = f_i, \quad C_2 f_0 + \frac{1}{3N\sigma_t}\frac{\partial f_0}{\partial x}\bigg|_{x=d} = 0. \end{aligned} \tag{7}$$

Here m and M are the masses of an electron and of an atom, respectively; $\nu_k = Nv^2\sigma_k$; $\nu_i = Nv^2\sigma_i$; $\nu = \nu_i + \sum_k \nu_k$; σ_k and ε_k are the cross section and threshold of excitation of the k-th level; σ_i and I are the ionization cross section and ionization potential; and, it is assumed that after an act of ionization the energy of the primary electron is equal to that of the secondary electron. A finite-difference scheme which gave a second-order approximation in the spatial variable and a first-order approximation in the energy variable was employed to solve Eq. (7). The following system of linear equations with a triangular matrix is obtained:

$$-a_{ij}f_{ij+1} - b_{ij}f_{ij-1} - c_{ij}f_{i+1j} + p_{ij}f_{ij} = \psi(f_{n>i,j}), \tag{8}$$

where the first index corresponds to W and the second index corresponds to x. Using the additional boundary condition for high energies $f_{i_0 j} = 0$, the system of algebraic equations (8) can be easily solved by the sweep method, which is stable ($p_{ij} \geqslant a_{ij} + b_{ij} + c_{ij}$, $a_{ij} >$

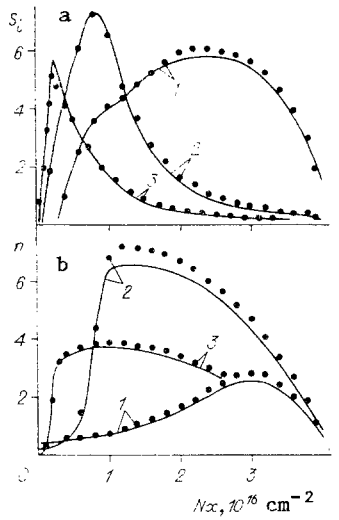

Fig. 1

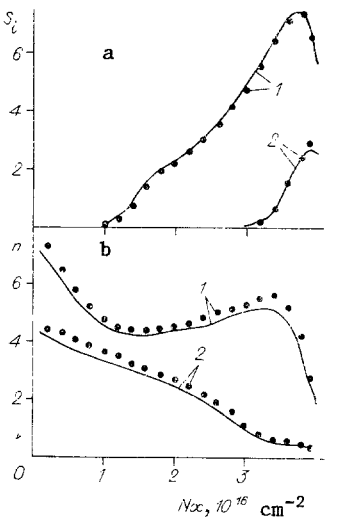

Fig. 2

TABLE 1

Nl, 10^{16} cm^{-2}	K				N_i/K				N_i/K				V_a, 10^8 cm/sec			
4,0	0,71	0,67	0,86	0,52	2,44	2,46	2,45	2,44	1,59	1,62	1,65	1,63	2,17	2,23	1,66	1,94
2,0	0,81	0,76	1,08	0,67	2,45	2,41	2,41	2,40	0,97	1,02	1,07	1,04	2,64	2,75	1,93	2,26
1,0	0,88	0,84	1,30	0,82	2,22	2,14	2,13	2,13	0,51	0,51	0,56	0,54	3,20	3,41	2,25	2,64
0,5	0,93	0,90	1,50	0,96	2,06	1,94	1,93	1,93	0,25	0,25	0,29	0,27	3,91	4,12	2,53	2,96
0,25	0,96	0,94	1,64	1,06	1,99	1,83	1,83	1,83	0,14	0,14	0,17	0,16	4,51	4,77	2,71	3,16
0,125	0,97	0,96	1,73	1,13	1,92	1,77	1,77	1,77	0,10	0,10	0,12	0,11	4,99	5,25	2,81	3,26

0, $b_{ij} > 0$, $c_{ij} > 0$). First the solution with $i = i_0 - 1$ is found, then the solution with $i = i_0 - 2$ is found, etc. It is easy to see that the method of solution is noniterative and requires the use of only one two-dimensional array f_{ij}. This last circumstance is quite important, since the algorithm was implemented on a BÉSM-6 computer, whose chief drawback in this case consisted of the fact that its working memory is limited. The results presented below were obtained primarily on a 150 × 130 grid. The error of the solution was equal to 1-2%, and the characteristic computing time for one variant was equal to ~10 sec. For comparison we note that approximately 1 h was required to solve the complete kinetic equation by the Monte Carlo method with approximately the same statistical error ($N_i \sim 2 \cdot 10^4$).

Numerical Results. The electron scattering cross sections of neon atoms were taken from [9]. The collection of electron levels was replaced by an effective level with an exciton energy $\varepsilon_1 = 16.6$ eV. To study the correctness of the TA the full kinetic equation was solved by the Monte Carlo method using the technique of zero collisions. The energy distribution of the injected electrons was taken in the form

$$f_i(0 < \varepsilon < \varepsilon_1) \sim \sin(\pi\varepsilon/\varepsilon_1)/\sqrt{\varepsilon},\ f_i(\varepsilon > \varepsilon_1) = 0,$$

which qualitatively describes the experimental data on ion–electron emission [10]. The interelectrode voltage drop was equal to 100 V. Both a uniform field $E(x) = E_0$ and a linear distribution of the field in the region near the cathode

$$E(x \leqslant l) = \frac{2u_1}{l}\left(1 - \frac{x}{l}\right) + E_1,\quad E(x > l) = E_1 \tag{9}$$

and in the region near the anode

$$E(x < d - l) = E_1,\quad E(x > d - l) = \frac{2U_1}{l}\left(1 - \frac{d}{l} + \frac{x}{l}\right) + E_1 \tag{10}$$

were studied. For the fixed values $U_1 = 80$ V, $E_1/N = 5 \cdot 10^{-16}$ V·cm^2, the characteristic scale of the nonuniformity of the field $N\ell = (0.125\text{-}4) \cdot 10^{16}$ cm^{-2} was varied. For comparison we note that in neon the mean free path length λ is virtually constant and for $\varepsilon = \varepsilon_1$, $N\lambda \approx 0.35 \cdot 10^{16}$ cm^{-2}.

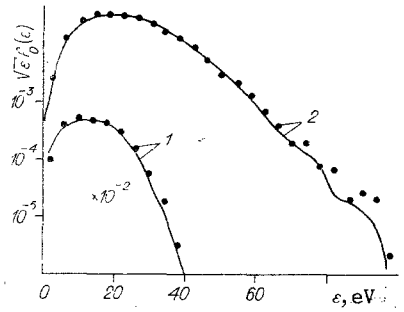

Fig. 3

The effect of the choice of boundary conditions on the error of the TA is indicated by the data presented in Table 1, where K is the transmission coefficient of the electrons, i.e., the ratio of the electron current at the cathode to the emission current; N_i is the number of ionization acts per injected electron; V_a is the average velocity of the electrons at the anode: the first value represents the results of the Monte Carlo calculations, the second value corresponds to the TA with the boundary conditions (6), the third value corresponds to the TA with Marshak's conditions, and the fourth value corresponds to the TA with Mark's conditions. The second and third columns refer to the field profile (9) and the last two columns refer to (10). As κ is increased, Marshak's and Mark's conditions give too low a value for V_a and too high values for K and N_i. For the value of N_i/K presented the error of the TA is significantly smaller. The boundary conditions (6) give good accuracy in calculating the values of N_i, which primarily determines the characteristics of the anode and cathode layers (even with $\ell < \lambda$).

The spatial behavior of different functionals of the EEDF also agrees with the exact solution. Figures 1 and 2 show the profiles of the ionization rate S_i (a) and the electron density n (b) in relative units for a nonuniform distribution of the field in the cathode and anode regions, respectively (curves 1-3 are for $N\ell$ = 4; 1; $0.25\cdot10^{16}$ cm^{-2}, the solid line refers to the TA with the boundary conditions (6), and the dots refer to the calculation by the Monte Carlo method). It is of interest to analyze the EEDF on the surface of the absorbing anode from the viewpoint of the problems of plasma diagnostics and such an analysis was performed in [11] for argon with the help of the Monte Carlo method. In this paper, we call attention to the good agreement (Fig. 3) between the TA with the boundary conditions (6) and the solution of the complete kinetic equation (dots) in a uniform electric field (the curves 1 and 2 are the EEDF normalized to unity at the anode with $E_0/N = 5\cdot10^{-16}$ and $2\cdot10^{-15}$ $V\cdot cm^2$).

The use of the TA with the generally accepted boundary conditions of Marshak and Mark leads in many cases to a large error in the calculation of the EEDF in the regions near the electrodes. However even a very simple modification of the boundary conditions permits reducing the error of the TA to 5-10%, which is fully acceptable from the viewpoint of the reliability of the scattering cross sections, employed in the calculations of the EEDF. We note that the statistical error of the Monte Carlo method significantly complicates its use in solving self-consistent problems, and in these cases the two-term approximation can be recommended.

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PENETRATION OF INTENSE PULSED MAGNETIC FIELDS INTO A CONDUCTOR

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The penetration of a pulsed magnetic field into an incompressible conductor is treated with consideration of Joulean heat liberation. Solutions are obtained for the case of penetration of a strongly decreasing magnetic field (significantly exceeding the saturation threshold) into a conductive semispace with planar boundary at constant specific heat and thermal conductivity. It is shown that consideration of the effect of bias current, where the limiting magnetic field is specified in the form of a step function, is of principle significance as regards both surface heating of the conductor and maintenance of intense magnetic fields in experimental equipment with planar boundaries.

It is well known [1, 2] that penetration of an intense magnetic field H(x, t) into a planar incompressible conductor (x > 0) can be described by the equations (in MKS units):

$$-\partial H/\partial x = j + \varepsilon_0\varepsilon_R\partial E/\partial t,\ \partial E/\partial x = -\mu_0\mu_R\partial H/\partial t,\ j = \sigma E,$$
$$\partial Q/\partial t = j^2/\sigma - \partial q/\partial x,\ q = -\lambda\partial\theta/\partial x - \tau_0\partial q/\partial t,\ Q = c_V\theta, \quad (1)$$

where j(x, t) is the volume conduction current density; E(x, t) is the electric field strength; $\varepsilon_0 = 8.85\cdot10^{-12}$ A·sec/(V·m); $\mu_0 = 4\pi\cdot10^{-7}$ V·sec/(A·m); μ_R, ε_R are the relative permutivities, which we assume constant (with either $\mu_R = \varepsilon_R = 1$, or $\mu_R = 1$, $\varepsilon_R = 0$, if we neglect displacement current as compared to conduction current); σ = const is the conductivity of the medium; Q(x, t) is the increment in heat content relative to the state at 0°C; q(x, t) is the thermal flux density; θ(x, t) is the conductor temperature; λ is the thermal conductivity coefficient; τ_0 = const is the thermal flux relaxation time; c_V is the specific heat of the conductor.

We will note that if the characteristic thermal flux relaxation time is large in comparison with the relaxation time τ_0, then $q/\tau_0\partial q/\partial t \gg 1$ and the fifth equation of Eq. (1) transforms to the usual Fourier law $q = -\lambda\partial\theta/\partial x$. And if the thermal flux changes significantly more rapidly than relaxation occurs, then $\partial q/\partial t \gg q/\tau_0$ and the fifth expression of Eq. (1) takes on the form

$$\frac{\partial q}{\partial t} = -\frac{\lambda}{\tau_0}\frac{\partial\theta}{\partial x}. \quad (2)$$

Neglecting displacement current in comparison to conduction current and taking $\tau_0 = 0$, we consider the process of magnetic field penetration into the conducting semispace x > 0 with the following boundary and initial conditions:

$$H(0,\ t) = H_0,\ q(0,\ t) = 0 \quad (t > 0); \quad (3)$$
$$H(x,\ 0) = 0,\ Q(x,\ 0) = 0 \quad (0 < x < \infty) \quad (4)$$

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